Enrollment No: ______________________ **Exam Seat No:**______________________

C. U. SHAH UNIVERSITY
Winter Examination-2022

Subject Name: Linear Algebra- I

Subject Code: 4SC03LIA1 **Branch: B.Sc. (Mathematics)**

Semester: 3 **Date: 23/11/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If V is Vector Space then which of the following is not true? (01)
For $\forall\alpha,\beta\in R$ and $\overline{x},\overline{y}\in V$
$(a).\propto\overline{x}\notin V$ $(b).\overline{x}+\overline{y}\in V$
$(c).\propto(\overline{x}+\overline{y})=\propto\overline{x}+\propto\overline{y}$ $(d).(\propto\beta)\overline{x}=\alpha(\beta\overline{x})$

b) Set $A=\{\overline{v}_1,\overline{v}_2,\dots,\overline{v}_n\}$ is base of vector space V then (01)
(a). A is linearly independant (b). A is linearly dependent
(c). $SPA=V$ (d). both (a) and (b)

c) Every _________ set of vector space can be extended to form a basis. (01)
(a). linearly independant (b). linearly dependent
(c). *empty* (d). non empty

d) V is vector space and dimV $=4$. If A is basis of V then number of vectors in A are ____ (01)
(a). 4 (b). 3 (c). 2 (d). 5

e) If $T:U\to V$ be any linear transformation, then (01)
(a). $N_T\subset$ U (b). $N_T\subset$ V (c). $N_T\subset R_T$ (d). None of these

f) A Linear transformation $T:U\to U$ is called____ (01)
(a). linear operator (b). linear functional
(c). both (a) and (b) (d). None of these

g) If $u\in R^3$ is an Euclidean space, $u=<2,1,-1>$ then $\|u\|=$ _____. (01)
(a). 6 (b). 5 (c). $\sqrt{6}$ (d). $\sqrt{5}$

h) If $T(x,y)=(x-y,2x-y,3x-y)$ then find $T(1,2)$. (01)

i) True or false: Identity matrix is Linearly Independent. (01)

j) True or false: Union of two subspace of vector space V is also subspace of vector space V. (01)

k) Define: Linear Transformation. (02)

l) Define: Inner product space. (02)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

A Let $V = \{(x, y)/x > 0, y > 0, x, y \in R\}$ and $(a, b), (c, d) \in V$ $(a, b) + (c, d) = (ac, bd)$ and $\propto (a, b) = (a^{\propto}, b^{\propto})$ Check whether V is vector space. **07**

B Prove that Intersection of two subspaces W_1 and W_2 of a vector space V is also a subspace. **05**

C Show that the set $A = \{(1, -2,3), (4,11,9), (-3,6, -9)\}$ is Linearly Depndent. **02**

Q-3 **Attempt all questions** **(14)**

A Prove that the set $A = \{(1,2,1), (2,1,0), (1, -1,2)\}$ forms a basis of R^3. **05**

B Express $v = (3,4,6)$ is a linear combination of $\bar{v}_1 = (1, -2,2), \bar{v}_2 = (0,3,4)$ and $\bar{v}_3 = (1,2, -1)$ **05**

C If S is non empty sub set of vector space V then show that $SP\ S$ is sub space of V. **04**

Q-4 **Attempt all questions** **(14)**

A Show that the set $\{\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n\}$ is linearly dependent iff $\exists$ a vector $\bar{v}_k$, $2 \le k \le n$, which is linear combination of its preceeding vectors $\bar{v}_1, \bar{v}_2, \dots, \bar{v}_{k-1}$. **05**

B Let $T: U \to V$ be any linear transformation. Then prove that Range of T $R_T = \{T(u)/u \in U\}$ is subspace of V. **05**

C Check whether $T: R^3 \to R^3$ defined as for $\forall (x, y, z) \in R^3$, $T(x, y, z) = (x - y, y - z, z - x)$, is a Linear Transformation **04**

Q-5 **Attempt all questions** **(14)**

A For a linear transformation $T: R^2 \to R^3$ defined as $T(x, y) = (x, x + y, y)$ $\forall (x, y) \in R^2$, find R_T, $N_T, r(T), n(T)$. **05**

B Let $T: R^2 \to R^2$; $T(x, y) = (x, -y)$; $\forall (x, y) \in R^2$ and let $B_1 = \{(1,0), (0,1)\}$ & $B_2 = \{(1,1), (1, -1)\}$ be two basis of R^2 then find $[T;\ B_1, B_2]$. **05**

C Find the linear transformation $T: R^3 \to R^2$ such that $T(e_1) = (1,1), T(e_1 + e_2) = (1,0), T(e_1 + e_2 + e_3) = (1, -1)$. **04**

Q-6 **Attempt all questions** **(14)**

A State and prove Rank-Nullity Theorem. **07**

B Let $u = (u_1, u_2), v = (v_1, v_2) \in R^2$. Define $< u, v >= 4u_1v_1 + u_2v_1 + 4u_1v_2 + 4u_2v_2$ then prove that $(R^2, < \cdot, \cdot >)$ is an inner product space. **05**

C Let R^4 be the Euclidean inner product. Find the cosine of the angle between the vectors $u = (1,0,1,0), v = (-3, -3, -3, -3)$ **02**

Q-7 **Attempt all questions** **(14)**

- **A** If W_1 and W_2 be two subspace of finite dimensional vector space V then prove that $\dim(W_1 + W_2) = dimW_1 + dimW_2 - \dim(W_1 \cap W_2)$ **07**
- **B** Find cosine angle between $u = (1,2)$ and $v = (0,1)$, also verify Cauchy-Schwarz Inequality . **04**
- **C** Verify Pythagorean theorem for vectors $u = (3,0,1,0,4,-1)$ and $v = (-2,5,0,2,-3,-18)$. **03**

Q-8 **Attempt all questions** **(14)**

- **A** If $u = (u_1, u_2, u_3), v = (v_1, v_2, v_3) \in R^3$, Define $< u, v >= 2u_1v_1 + u_2v_2 + u_3v_3$ then prove that $< u, v >$ is an inner product space on R^3. **06**
- **B** Check whether set $W = \{(x, y, z)/x - 2y + 3z = 0; x, y, z \in R\}$ is subspace of R^3. **05**
- **C** Define: (i). Linear Independence, (ii) Linear span of a Set **03**